

CAUGHT IN THE MIDDLE: TENSIONS RISE WHEN TEACHERS AND STUDENTS RELINQUISH ALGORITHMS

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Relinquishing algorithms in favour of less-conventional approaches for calculating is a challenge for both students and teachers. Analysis of a Roundtable Reflection session with Grade 3 and Grade 4 teachers reveals that tensions for teachers arise: when students automatically choose to use algorithms rather than alternative approaches; when they contemplate their class's transition to the next teacher; and when teachers think about parent/community/curriculum expectations concerning calculation processes. It is essential to address these tensions to sustain desired changes in teaching practice.

INTRODUCTION

Life in the twenty-first century requires an unprecedented level of mathematical knowledge and skill for full participation in community life, and for access to opportunities in education and employment. It is therefore of concern that recent research suggests that some Australian students under-perform in mathematics despite consistent attempts to improve mathematics learning and teaching. For example, Gervasoni, Turkenburg, & Hadden (2007) found that 30% of students beginning their final year of primary school in regional Victoria have underdeveloped arithmetic reasoning strategies, a key indicator of mathematical competence. Improving mathematics learning and teaching in the primary school is still necessary.

This paper reports on one aspect of a research program that overall seeks to discover whether a supported professional learning approach using *Self-Study* as a methodology and *empty number lines* as a catalyst for improving mental computation will challenge conventional mathematics learning and teaching and result in better learning outcomes for students in Grade 3 and Grade 4 that are sustained throughout the primary school. The aspect considered here are the tensions that arise when teachers delay teaching algorithms until later in the primary school and instead emphasise mental computation and subsequent recording of associated reasoning on empty number lines. Insight about these tensions is provided through analysis of themes emerging during a Roundtable Reflection session involving six teachers and two researchers. Revealing these tensions provides important insight for those engaged in designing professional learning programs and new curriculum that aims to improve children's arithmetic reasoning and associated teaching practice. Anticipating the tensions that teachers may experience when this is attempted, and knowing the type of support that teachers and students may require will assist new approaches to be more effective and sustainable.

CONSIDERATIONS FOR CHANGING TEACHER PRACTICE

Teachers are often reluctant to identify and alter habitual ways of teaching. Consequently, new curriculum policy may not transfer to changes in teaching practice. This issue is important to address when introducing an innovation that involves delaying the teaching of algorithms in favour of greater emphasis on mental computation. The concepts underpinning the research reported in this paper (Self-Study, Roundtable Reflection and empty number lines) seek to address this issue.

Self-Study and Roundtable Reflection

Self-Study methodology (Berry & Loughran, 2002; Brandenburg, 2008; Bullough & Pinnegar, 2001; Hamilton & Pinnegar, 1998; Russell, 1999) and Roundtable Reflection (Brandenburg, 2008) are features of this research. Self-Study has emerged from the action research/reflective practice/ethnographic research traditions, and is an important development in teacher education research (Zeichner, 1999). An expectation of Self-Study is that teachers identify the issues and tensions they experience and unpack these using systematic reflection as a means of understanding more deeply the ways that their teaching impacts on student learning. Teachers theorise their learning through ongoing data analysis and through experiencing changes in understanding that demands enactment in practice of new learning (Loughran, 2006). Learning is named and scrutinised during Roundtable Reflections.

Roundtable Reflection (RR) involves structured pedagogical inquiry that aims to identify and challenge assumptions (Brookfield, 1995) which more often than not, are taken-for-granted. Fundamental to the success of the RR approach is the establishment of a secure, trusting environment where each participant's voice is acknowledged and respected. Each RR begins by teachers identifying and briefly describing a critical incident/interaction that has occurred in their classroom. The group then prioritises, and determines one incident to be scrutinised in detail. Focused questioning and subsequent challenging of assumptions about learning and teaching enables some confrontation and de-stabilisation of practice (Segall, 2002).

Using "empty number lines" as a catalyst for improving arithmetic reasoning

The mathematical focus for this research is mental computation and arithmetic reasoning strategies, which have been the focus of many studies (e.g., Clarke, Cheeseman, Gervasoni, et al., 2002; Fuson, 1992; Gervasoni, 2006; Steffe, Cobb, & von Glasersfeld, 1988). Not all children have these strategies available or choose wisely to fit the characteristics of a strategy to the demands of a task (Griffin, Case, & Siegler, 1994). Narode, Board, & Davenport (1993) suggest that introducing algorithms too early in schooling is detrimental to students' developing arithmetic reasoning strategies. However, it is common for Australian teachers to introduce Grade 2 and Grade 3 students to algorithms for addition and subtraction. This is contrary to curriculum guidelines that no longer deem this necessary (Victorian Curriculum and Assessment Authority, 2008). Indeed, it is now broadly accepted that the conventional focus on taught procedures for calculating can negatively impact on

children's number sense (Clarke, Clarke, & Horne, 2006). It is proposed that a more effective approach is to delay the introduction of algorithms and instead focus on mental computation. An associated practice is to encourage students to record their thinking on empty number lines, so that monitoring and reflection on strategy choice may occur. This approach is widely used in the Netherlands and aims to link early mathematics activities to students' own informal counting and structuring strategies (Beishuizen & Anghileri, 1998). Monitoring strategy selection by written recording on empty number lines and through reflection on strategy choice in class discussion is essential to stimulate contraction towards higher-level strategies (Beishuizen & Anghileri, 1998). This approach seldom occurs in Australia.

TENSIONS WHEN TEACHERS LET GO OF ALGORITHMS

The research reported in this paper involved trialling a curriculum reform in which the teaching of algorithms for Grade 3 and Grade 4 students was delayed in favour of emphasising mental computation, with students' reasoning strategies recorded on empty number lines to enable monitoring and reflection of strategy choice. The research involved six teachers from two schools in a low-socio-economic area of a regional Victorian city. The curriculum reform aimed to improve students' arithmetic reasoning strategies. Assessment of students in the region at the beginning of 2008 using the Early Numeracy Interview (Clarke, Sullivan, & McDonough, 2002) showed that many of the students were using counting-based strategies rather than reasoning strategies to solve the following problems: $4+4$; $2+19$; $4+6$; $27+10$; and $10-7$ (44% of Grade 3 students, $n=584$, $N=1314$; and 22% of Grade 4 students, $n=282$, $N=1261$).

The research approach emphasised teacher professional learning using Self-study and Roundtable Reflection. Teachers met with the researchers for five professional learning sessions (June to November 2008) that considered the theoretical underpinning the approach, teaching advice and curriculum planning. An important part of each session was a Roundtable Reflection (RR) that was recorded, transcribed and analysed by the researchers. The data reported in this paper relate to the fourth Roundtable Reflection (RR4), conducted in Term 3 as part of a 2-hour professional learning session. The RR began with each teacher describing a critical incident that had occurred recently during a mathematics lesson, and then the selection of one critical incident by the group for detailed discussion and inquiry. For example, the critical incident presented by Sarah in RR4 follows.

Sarah What I have found is that my top group automatically go to vertical addition [algorithm]....

Meg Some of mine do too!

Sarah They don't even attempt number lines ... they are quite happy right from the start just to quickly do vertical stuff and some of the little ones try to do it but then they are getting confused with it. So actually then we go back to the number line and they can see it so much more clearly.... Do you just let those kids go now, those top groups? Because basically....

you can't really try and teach them [to use number lines instead] can you?
Because if they have been doing it [algorithms] since Grade 1 or 2....

The group chose this critical incident for detailed discussion and inquiry. It reflected a tension that most of the teachers were experiencing and were searching to resolve. Letting go of algorithms was proving difficult for both teachers and students.

Sally I would like to work on Sarah's because ... I am finding that too. That there are kids that just want to do the vertical computation and.....

Meg I do too.

Sally And that's what they want to do and that's what they are good at. I think some of the kids think they are going backwards if they don't do it that way [vertical method]

Meg I'm finding that some of mine aren't as good at it [algorithms] as they think so they are making mistakes but I am not re-teaching it to them. I am not saying well, you know, remember this is where you trade....

Even though the tension associated with letting go of algorithms is clear in this excerpt, Meg's final statement acknowledges that some children really don't understand algorithms, a key reason for the approach being trialled in this research.

Key themes in the dialogue extract

Once RR4 was transcribed, the researchers identified the dialogue in the transcript that dealt with the critical incident chosen for discussion and inquiry. Seventy-four pieces of dialogue were extracted with each providing substantial comment about the critical incident. Each piece of dialogue was examined and any emerging themes identified. This process was repeated and the themes confirmed or refined. Table 1 presents the eight most commonly occurring themes identified in the extract.

Key Themes in the Dialogue	Occurrences
Children don't understand algorithms	11
Children are not letting go of algorithms	7
Using number lines for proof and for understanding	5
Timing of introducing algorithms	5
Community/parent/government expectations	5
Habitual use of algorithms or children automatically using algorithms when presented with calculations	5
Children don't understand number concepts	5
Transition to the next class/teacher	5

Table 1. Key themes in the dialogue extract

These themes highlight that some children have difficulty understanding algorithms, but also that once taught some children have difficulty relinquishing algorithms. In contrast, teachers believed that using number lines aided children's understanding and the ability to prove their calculations. Tensions for the teachers also centred on community expectations, transition to the next class and teacher, and timing for the introduction of algorithms. Some examples of the dialogue follow.

Children don't understand algorithms

- Jan When I introduced the algorithms to grade two, because they [the curriculum] said I should ... the majority of children were very mixed up and some of the answers they were getting showed they had absolutely no concept ... [that] sixty four and twenty one couldn't possibly be a thousand and something, but that is what they were writing. But since they have been using the number line, they have really increased their understanding. In fourth term, I don't know whether we should perhaps try some algorithms and see has there been a transfer of knowledge.

This extract highlights that students often do not understand algorithms or the answers produced. A further tension is revealed; even though the teacher explains that using the number line had been successful in building children's understanding, she wonders whether algorithms should be attempted again in Term 4. This is likely due to her contemplating the expectations of the students' new teacher.

Using number line as proof and to build understanding

- Sarah One of my little ones the other day was trying to do the algorithm, which wasn't right, so then we actually went back to the number line to actually explain what she was doing.
- Researcher And what happened then?
- Sarah She could see where she had gone wrong and everything with it.

This extract highlights that the teacher chose to build the student's understanding through instruction using the number line. She also noted that students continue to use algorithms even though they may not understand them or recognise whether answers are correct. This relates also to the first and second themes in Table 1.

Timing of introducing algorithms

- Leini Sometimes you see them write answers to an algorithm that are like eight hundred above what it should even be because they don't even know [understand] the quantity.
- Sally They don't even know what this is.
- Meg Well that came out in the early numeracy testing didn't it (all agree) ... the lower numbers - they are fine, but when they had to estimate [numbers] in between [for larger numbers]....
- Sally [When] I have been teaching f{Grade} 5 and 6, I say, "now have a look at the sum, do it, have a look at your answer, is it reasonable?" You know

and that kind of questioning as you are doing algorithms.... But it is also the timing and the scope and sequence [that] is important. But we aren't starting it off [using number lines] from the very beginning so we are caught in the middle of kids who already know how to do it [algorithms]—... and they haven't had what we would like them to have [had] to get there [to the understanding of quantity].

This extract reveals an important tension for the teachers. Even though this research anticipated delaying the introduction of algorithms, in fact the students had already encountered algorithms in earlier classes. The teachers felt caught in the middle because they could not undo this situation, but had to deal with students who didn't have the number sense necessary to understand or successfully use algorithms.

Transition to the next class/teacher

Jan I'm sure the Grade 3 teachers would like them to have done some vertical addition and so there is my dilemma.

Another example:

Sally What happens if we have decided we are *out with algorithms*, and you know and it's great and there is sound philosophy behind it and we are all for it! We go to a Grade 5 teacher who goes, "This kid doesn't know how to do a division sum." You know it's that whole change, shifting.

Both examples provide insight about the tensions encountered when teachers contemplate the expectations of the next teacher. Tensions surrounding transition to the next class highlight the need for a whole school approach to curriculum change.

DISCUSSION AND CONCLUSION

Key themes that emerged from analysis of this Roundtable Reflection excerpt were: children not understanding algorithms; children not letting go of algorithms once they had encountered them; the usefulness of number lines for providing proof and for understanding computational reasoning; the timing of introducing algorithms; tensions associated with community/parent/government expectations; children's habitual use of algorithms or children automatically using algorithms when presented with calculations to perform; children not understanding number concepts involving 2-digit numbers; and tensions related to transition to the next class/teacher.

These themes provide insight about the tensions teachers face when they are implementing and sustaining curriculum change and the difficulties teachers and children have in relinquishing algorithms in favour of mental computation. We recommend that professional learning programs and school and system leaders support teachers by assisting them to identify and address the tensions that arise.

Overall, it seems that teachers in this research were *caught in the middle* between research-based innovative practice and the *tug* of more conventional practice. These Grade 3 and Grade 4 teachers were also *caught in the middle* between curriculum in the junior primary school that focused on mental computation, but also introduced

vertical methods of addition and subtraction, and expectations in the senior primary school that calculation methods would be algorithm-based. One important theme that the teachers explored during the Roundtable Reflection was the optimal timing for introducing algorithms. They believed their students had encountered algorithms too early, and that this had impeded attempts to develop students' mental computation strategies. They proposed that Grade 5 was soon enough to introduce algorithms.

Meg If [students] have got a good understanding of mental computation shouldn't it, when we go to teach them the algorithm, not be a problem?

Leini Well that's, I suppose, the theory.

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